Longitudinal Beam Physics on UMER

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Outline

• Importance of longitudinal structure
• Gun physics
• Launching waves
• Speed of sound
• Beam transport
Real electron beam distributions are not named after famous people!
**Problem Statement**

**Problem:**
Real beams have *unwanted* velocity (energy) or density modulations

**Reasons:**
- Drive laser fluctuations in photoinjectors
- Over-focusing or under-focusing in longitudinal focusing systems

**Technique:**
- Introduce perturbation "*deliberately*" in an intense beam and study its evolution.
- **UMER** provides the platform for such experiments
## Sources of problems

<table>
<thead>
<tr>
<th>What</th>
<th>Where</th>
<th>How</th>
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<tbody>
<tr>
<td><strong>Space charge (self) fields</strong></td>
<td>electron source, low energy injector/linac,</td>
<td>Space charge converts density modulation to energy modulations, and causes time dependant defocusing</td>
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<tr>
<td><strong>Wakefields/Higher OM</strong></td>
<td>Cavities and structures</td>
<td>Bunch structure can excite high frequency modes</td>
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<tr>
<td><strong>Coherent Synchrotron Radiation (CSR)</strong></td>
<td>bends</td>
<td>Bunch structure can excite coherent synchrotron radiation</td>
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Two Types of Longitudinal Problems on UMER

- Longitudinal effects in the gun and space charge driven instabilities
- Longitudinal effects in in multi-turn transport: evolution of perturbations

These are relevant to other accelerators
1. Longitudinal structure and space charge instabilities in the gun
Space-charge driven longitudinal beam breakup
Child- Langmuir Limit Revisited

Conventional Child-Langmuir Analysis:
Uniform 1-D current distribution
Steady state
Pulse length $\tau_p >>$ Electron transit time $T$

Real Photoinjector Analysis:
non-uniform 3-D time-dependant current distribution
transient state
Pulse length $\tau_p <$ Electron transit time $T$
Cathode – Anode Transit Time ($T$) in guns

**Non-relativistic**

\[ T = \sqrt{\frac{2D^2m}{eV_0}} \]

- $D = A/C$ gap = 2.5 cm in UMER
- $V_0 =$ Gun voltage = 10 kV

For UMER gun $T \approx 1$ ns

**Relativistic**

\[ T = \frac{mc\gamma_f\sqrt{1 - \frac{1}{\gamma_f^2}}}{eE_A} \]

- $\gamma_f =$ relativistic factor at gun exit $\approx 3$ for 1 MeV
- $E_A =$ average applied electric field $\approx 30$ MV

For Relativistic gun $T \approx 0.15$ ns

For a typical RF photoinjector $\tau_p \approx 0.01$ ns
Experimental data on charge per pulse vs drive laser intensity. The graph shows the relationship between total charge in current pulse and relative laser intensity for a 5-kV gun. 

Critical Current \( (J_{\text{crit}}) \) in short pulse mode for onset of space charge instabilities

Critical current density:

\[
J_{\text{CRIT}} = 2 \sqrt{1 - \frac{3}{4} X_t^2} J_{CL}
\]

\[
X_t = \frac{\tau_p}{T}
\]

Critical charge density:

\[
Q_{\text{CRIT}} = 2 \sqrt{1 - \frac{3}{4} X_t^2} Q_{CL}
\]

Short pulse mode \( X_t \ll 1 \)

\[
J_{\text{CRIT}} \approx \frac{3}{4 X_t} J_{CL}
\]

\[
Q_{\text{CRIT}} \approx \frac{3}{4} Q_{CL}
\]

UMER is a unique test bed for studying the evolution of current perturbations.
UMER Drive Laser Setup

E-beam

UV (355nm) Laser
FWHM = 5ns

Electron Gun

Nd:YAG Laser

KTP/BBO Crystals
Space Charge Instabilities in the UMER Gun

Perturbation **below** the critical current density

At large laser power

Perturbation splitting into sub pulses

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Jayakar Charles Tobin Thangaraj

Perturbation **above** the critical current density
Charge per pulse vs laser intensity
Onset of longitudinal instability
Experimental data

\[ Q_{\text{CRIT}} \approx \frac{3}{4} Q_{\text{CL}} \]

Critical Current Density vs. pulse length
Simulation

\[ J_{\text{CRIT}} = 2 \frac{1 - \sqrt{1 - \frac{3}{4} X_t^2}}{X_t^3} J_{CL} \]

\[ X_t = \frac{\tau_p}{T} \]

\[
\begin{align*}
\text{Normalized Current Density (} J_{\text{crit}}/J_{CL} \text{)} & \sim 10^{3} \\
\text{Normalized Pulse Length } [\tau_p/T_{\text{transit}}] & \sim 10^{-1}
\end{align*}
\]

2. Evolution of longitudinal structure in beam transport

Thanks to: Jayakar Charles Tobin Thangaraj
Experimental Results of Laser-induced Space Charge waves

UMER Top View

Near gun

At s = 5.75 m

Normalized Current

Time (ns)

Laser induced perturbation

Fast wave

Slow wave
Evolution of Multiple Pulses

Modulation observed to disappear, return, then start to disappear again as beam travels through UMER

Bergoz (62.6 cm)
BPM 0 (82.6 cm)
BPM 1 (194 cm)
BPM 2 (258 cm)
BPM 3 (323 cm)
BPM 4 (386 cm)
BPM 5 (450 cm)
BPM 6 (514 cm)
BPM 7 (578 cm)
BPM 8 (642 cm)
BPM 9 (706 cm)
BPM 10 (770 cm)
BPM 11 (834 cm)
BPM 12 (898 cm)

John Harris,
PhD dissertation 2005
https://drum.umd.edu/dspace/handle/1903/2906
Theory of Space Charge Waves
(1-D Cold fluid model)

Space charge line density $\Lambda(z,t) = \Lambda_0 + \Lambda_1 e^{i(\omega t - kz)}$

Velocity $v(z,t) = v_0 + v_1 e^{i(\omega t - kz)}$

Current $I(z,t) = I_0 + I_1 e^{i(\omega t - kz)}$
Derivation of Sound Speed

(one-dimension cold-fluid model)

Definition of perturbation

\[
\begin{align*}
\Lambda(z,t) &= \Lambda_0 + \Lambda_1 e^{i(\omega t - kz)} \\
v(z,t) &= v_0 + v_1 e^{i(\omega t - kz)} \\
I(z,t) &= I_0 + I_1 e^{i(\omega t - kz)}
\end{align*}
\]

Continuity equation

\[
\frac{\partial (\Lambda v)}{\partial z} + \frac{\partial \Lambda}{\partial t} = 0
\]

Momentum equation

\[
\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} = \frac{q}{\gamma_0 m} E_s
\]

Maxwell’s equation and boundary conditions

\[
E_s = -\frac{g}{4\pi \varepsilon_0} \left( \frac{\partial \Lambda}{\partial z} + \frac{1}{c^2} \frac{\partial I}{\partial t} \right)
\quad g = 2 \ln \frac{b}{a}
\]

Dispersion equation

\[
(\omega - kv_0)^2 - C_s^2 k^2 = 0
\]

\(C_s = \text{Sound speed}\)

Phase velocity of fast/slow waves

\[
\begin{align*}
v_f &= \frac{\omega}{k_+} = v_0 + C_s \\
v_s &= \frac{\omega}{k_-} = v_0 - C_s
\end{align*}
\]

\[
C_s = \sqrt{\frac{qg\Lambda_0}{4\pi \varepsilon_0 \gamma_0^5 m}}
\quad \text{or} \quad C_s = \sqrt{\frac{egI}{4\pi \varepsilon_0 m v_0 \gamma^5}}
\]

For UMER \(C_s \approx 10^6 \text{ m/s}\)
Evolution of Space-Charge Waves
Fast (Forward) and Slow (Backward) Waves

Definition of perturbation:

\[
\begin{align*}
    v_1(0,t) &= \delta v_0 p(t) \\
    I_1(0,t) &= \eta I_0 p(t) \\
    \Lambda_1(0,t) &= (\eta - \delta) \Lambda_0 p(t)
\end{align*}
\]

Assume pure density perturbation \( \delta = 0 \)

Algebraic equations of

\[
\Lambda_1(z,t) = \frac{\Lambda_0}{2} \eta \left[ h \left( t - \frac{z}{v_0 - C_s} \right) + h \left( t - \frac{z}{v_0 + C_s} \right) \right]
\]

\[
v_1(z,t) = \frac{C_s}{2} \eta \left[ -h \left( t - \frac{z}{v_0 - C_s} \right) + h \left( t - \frac{z}{v_0 + C_s} \right) \right]
\]

Red = slow (backward) wave
Blue = fast (forward) wave

\( h \) function is a wave that depends of the initial conditions etc.
Evolution of Space-Charge Waves

\[ \Lambda_1(z,t) = \frac{\Lambda_0}{2} \eta \left[ h \left( t - \frac{z}{v_0 - C_s} \right) + h \left( t - \frac{z}{v_0 + C_s} \right) \right] \]

\[ v_1(z,t) = \frac{C_s}{2} \eta \left[ -h \left( t - \frac{z}{v_0 - C_s} \right) + h \left( t - \frac{z}{v_0 + C_s} \right) \right] \]
Space Charge Wave Transport
Experimental Results from UMER
Experimental Results of Laser-induced Space Charge waves

UMER Top View

Laser induced perturbation

Near gun

At s = 5.75 m

$C_s (\text{exp}) = 1.54 \times 10^6 \text{ m/s}$

$C_s (\text{theory}) = 1.30 \times 10^6 \text{ m/s}$
Multiturn observation of waves in UMER
Multiturn observation of waves in UMER
Space Charge Waves & Instabilities

UV (355nm) FWHM = 5ns Nd:YAG Laser

At large laser power: explosive splitting in gun

At low laser power: gentle splitting in transport

At s = 5.75 m

Perturbation splitting into sub pulses

Fast wave, Slow wave

Laser induced perturbation
Space Charge Converts Density Modulation into Energy Modulations

Experiment at Brookhaven DUV FEL after perturbed beam is accelerated to 75 MeV

Jonathan Neumann, Dissertation 2005
https://drum.umd.edu/dspace/handle/1903/2437

Q = 0.16 nC
Is the inverse Humpty-Dumpty Effect possible?

Humpty Dumpty sat on a wall.
Humpty Dumpty had a great fall.
All the king’s horses and all the king’s men
Couldn’t put Humpty together again.

Illustration by John Tenniel From *Through the Looking-Glass.*